

Second degré - Correction de l'exercice 16

a).

$$\begin{aligned}
 4x^4 - 5x^2 + 1 = 0 &\iff 4X^2 - 5X + 1 = 0 && \text{avec } X = x^2 \\
 &\iff X = 1 \text{ ou } X = \frac{1}{4} \\
 &\iff x^2 = 1 \text{ ou } x^2 = \frac{1}{4} \\
 &\iff x = \pm\sqrt{1} \text{ ou } x = \pm\sqrt{\frac{1}{4}}
 \end{aligned}$$

Conclusion : $\mathcal{S} = \{-1; -1/2; 1/2; 1\}$.

b)

$$2x^4 - x^2 + 1 = 0 \iff 2X^2 - X + 1 = 0 \quad \text{avec } X = x^2$$

Conclusion : $\mathcal{S} = \emptyset$ (car $\Delta < 0$)

c)

$$\begin{aligned}
 x^4 - 8x^2 - 9 = 0 &\iff X^2 - 8X - 9 = 0 && \text{avec } X = x^2 \\
 &\iff X = -1 \text{ ou } X = 9 \\
 &\iff x^2 = -1 \text{ ou } x^2 = 9 \\
 &\iff x = \pm\sqrt{9}
 \end{aligned}$$

Conclusion : $\mathcal{S} = \{-3; 3\}$.

d). Soit $x \neq 0$,

$$\begin{aligned}
 4x^2 - 35 - \frac{9}{x^2} = 0 &\iff 4x^4 - 35x^2 - 9 = 0 \\
 &\iff 4X^2 - 35X - 9 = 0 && \text{avec } X = x^2 \\
 &\iff X = -\frac{1}{4} \text{ ou } X = 9 \\
 &\iff x^2 = -\frac{1}{4} \text{ ou } x^2 = 9 \\
 &\iff x = \pm\sqrt{9}
 \end{aligned}$$

Conclusion : $\mathcal{S} = \{-3; 3\}$.

e).

$$\begin{aligned}
 -2x^4 + 12x^2 - 16 = 0 &\iff X^2 - 6X + 8 = 0 && \text{avec } X = x^2 \\
 &\iff X = 2 \text{ ou } X = 4 \\
 &\iff x^2 = 2 \text{ ou } x^2 = 4 \\
 &\iff x = \pm\sqrt{2} \text{ ou } x = \pm\sqrt{4}
 \end{aligned}$$

Conclusion : $\mathcal{S} = \{-2; -\sqrt{2}; \sqrt{2}; 2\}$.

f)

$$\begin{aligned}
 x^4 + 5x^2 + 4 = 0 &\iff X^2 + 5X + 4 = 0 && \text{avec } X = x^2 \\
 &\iff X = -1 \text{ ou } X = -4 \\
 &\iff x^2 = -1 \text{ ou } x^2 = -4.
 \end{aligned}$$

Conclusion : $\mathcal{S} = \emptyset$.